Diploma Programme
Programme du diplôme
Programa del Diploma

Diploma Programme
Programme du diplôme
Programa del Diploma

**Mathematics: applications and interpretation**
**Standard level**
**Paper 1**

30 October 2023

**Zone A** afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

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| | | | | | | | | | |

1 hour 30 minutes

## Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.


8823 – 7214

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

**1.** [Maximum mark: 7]

Joel is a keen cyclist who keeps a record of his performance. The following table shows the time, in minutes, it takes him to ride one kilometre on hills with different gradients. The gradient of each hill is constant.

| **Gradient $G$ (%)** | 0 | 4 | 10 | 15 | 20 |
|---|---|---|---|---|---|
| **Time $T$ (min.)** | 2.11 | 5.39 | 10.56 | 13.20 | 18.58 |

(a) (i) Find the equation of the regression line of $T$ on $G$.

(ii) Describe the correlation between $T$ and $G$ with reference to the value of $r$, the Pearson's product-moment correlation coefficient. [4]

On Saturday, Joel intends to ride a hill with a gradient of $17\%$.

(b) Estimate the time it will take Joel to ride one kilometre up the hill. [2]

This morning, Joel rode one kilometre up a hill, and it took $22$ minutes.

(c) Explain why it would be inappropriate to use the equation found in part (a) to estimate the gradient of this hill. [1]

**(This question continues on the following page)**

**(Question 1 continued)**

**2.** [Maximum mark: 6]

The Great Pyramid of Giza is the oldest of the Seven Wonders of the Ancient World. When it was built, 4500 years ago, the measurements of the pyramid were in Royal Egyptian Cubits ($\text{REC}$).

[Source: Nina Aldin Thune. https://en.wikipedia.org/wiki/Great_Pyramid_of_Giza#/media/File:Kheops-Pyramid.jpg. Licensed under CC BY 2.5 https://creativecommons.org/licenses/by/2.5/#. Image adapted.]

Viktor reads online that $1\,\text{REC}$ is equal to $0.52$ metres, rounded to two decimal places.

(a) Write down the upper and lower bounds of $1\,\text{REC}$ in metres. [2]

The Great Pyramid of Giza has a square base with side lengths of $440\,\text{REC}$ and a height of $280\,\text{REC}$. Viktor assumes that these two measurements are exact and that the Great Pyramid can be modelled as a square-based pyramid with smooth faces.

(b) Find the minimum possible volume of the pyramid in cubic metres. [4]

**3.** [Maximum mark: 6]

Consider the graph of the following function:

$$g(x) = \frac{8}{x} + \frac{x^2}{2}\text{, for } x \neq 0.$$

(a) Write down the equation of the vertical asymptote of $g(x)$. [1]

(b) Find $g'(x)$. [3]

(c) Write down the interval in which $g(x)$ is increasing. [2]

**4.** [Maximum mark: 8]

On the following Voronoi diagram, the coordinates of three farmhouses are $\mathrm{A}(0, 3)$, $\mathrm{B}(8, 3)$ and $\mathrm{C}(8, 13)$, where distances are measured in kilometres. Each farmhouse owns the land that is closest to it, and their boundaries are defined by the dotted lines on **Diagram 1**.

**Diagram 1**

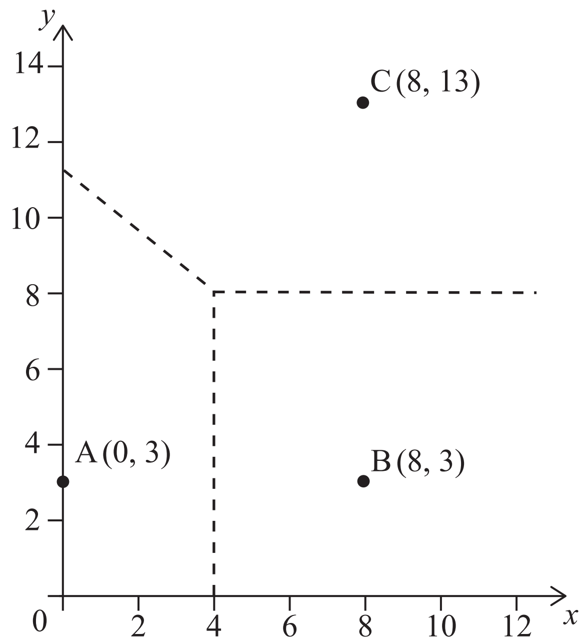
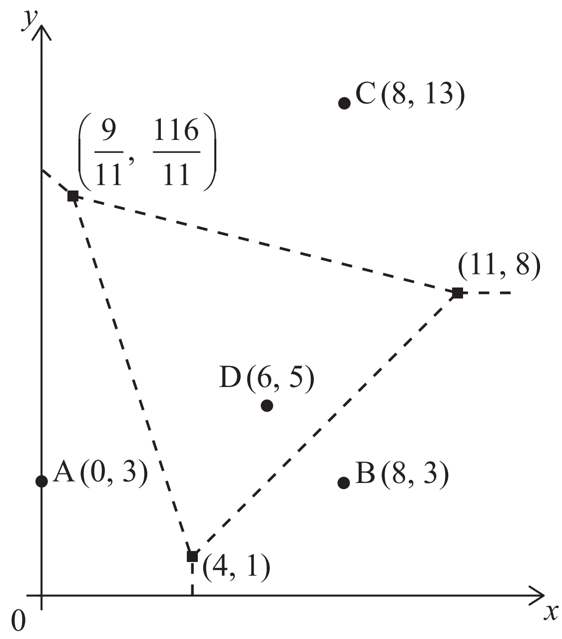



**Diagram 2**




To provide water to the farms it is decided to construct a well at the point where the boundaries meet on **Diagram 1**.

(a) Write down the coordinates of this point. [1]

(b) Find the equation of the perpendicular bisector of $[\mathrm{AC}]$. [3]

An additional farmhouse $\mathrm{D}(6, 5)$ is built on the land. The Voronoi diagram has been redrawn to show the new boundaries. The coordinates of the vertices of these boundaries are indicated on **Diagram 2**.

A wind turbine is to be built at one of the vertices.

(c) The wind turbine should be as far from the nearest farmhouses as possible.

(i) By calculating appropriate distances, find the location of the wind turbine.

(ii) Hence, write down the distance of the wind turbine to the nearest farmhouse. [4]

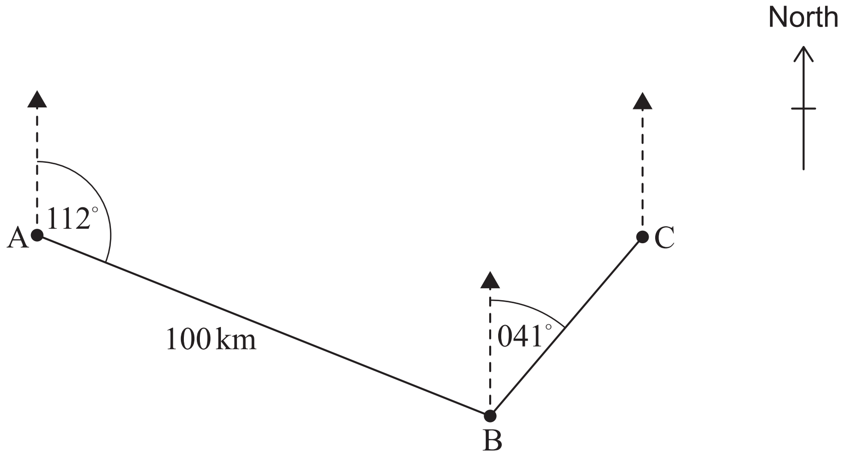
**(This question continues on the following page)**

**(Question 4 continued)**

**5.** [Maximum mark: 6]

Jason sails his boat from point A for a distance of $100\,\text{km}$, on a bearing of $112^\circ$, to arrive at point B. He then sails on a bearing of $041^\circ$ to point C. Jason's journey is shown in the diagram.

**diagram not to scale**




(a) Find $\text{A}\hat{\text{B}}\text{C}$. [2]

Point C is directly east of point A.

(b) Calculate the distance that Jason sails to return directly from point C to point A. [4]

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**6.** [Maximum mark: 4]

Consider the following function:

$$h(x) = \frac{2}{\sqrt{x-1}} + \frac{1}{2}\text{, for } x > 1\text{.}$$

(a) Find $h^{-1}(1)$. [2]

(b) Find the domain of $h^{-1}(x)$. [2]

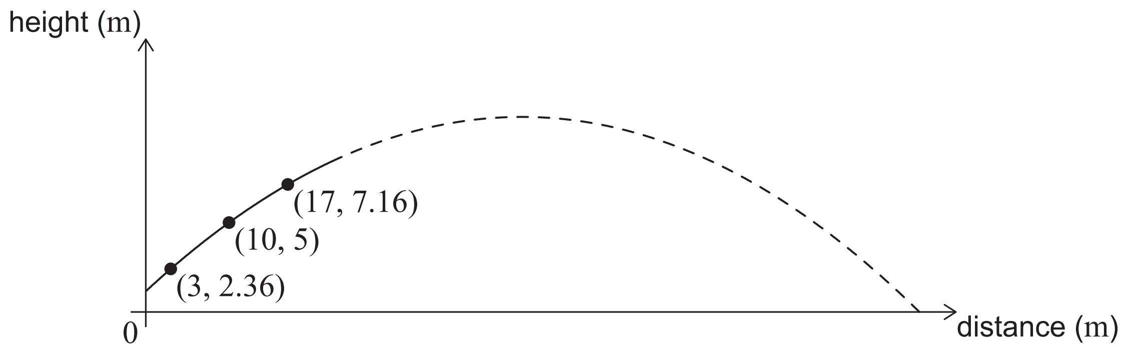
**7.** [Maximum mark: 9]

A sports player on a horizontal athletic field hits a ball. The height of the ball above the field, in metres, after it is hit can be modelled using a quadratic function of the form $f(x) = ax^2 + bx + c$, where $x$ represents the horizontal distance, in metres, that the ball has travelled from the player.

A specialized camera tracks the initial path of the ball after it is hit by the player. The camera records that the ball travels through the three points $(3, 2.36)$, $(10, 5)$ and $(17, 7.16)$, as shown in **Diagram 1**.

**diagram not to scale**

**Diagram 1**



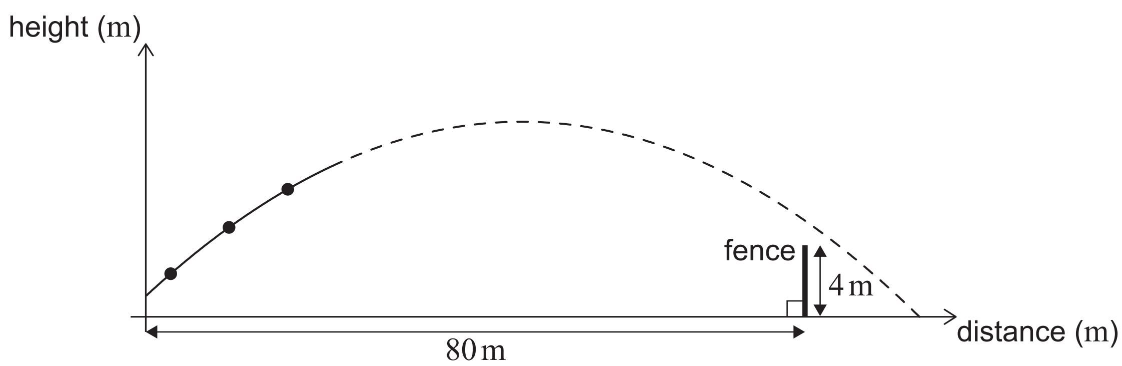

(a) Use the coordinates $(3, 2.36)$ to write down an equation in terms of $a$, $b$, and $c$. [1]

(b) Use your answer to part (a) and two similar equations to find the equation of the quadratic model for the height of the ball. [3]

A 4-metre-high fence is $80$ metres from where the player hit the ball, as shown in **Diagram 2**.

**diagram not to scale**

**Diagram 2**




(c) Show that the model predicts that the ball will go over the fence. [3]

(d) Find the horizontal distance that the ball will travel, from the player until it first hits the ground, according to this model. [2]

**(This question continues on the following page)**

**(Question 7 continued)**

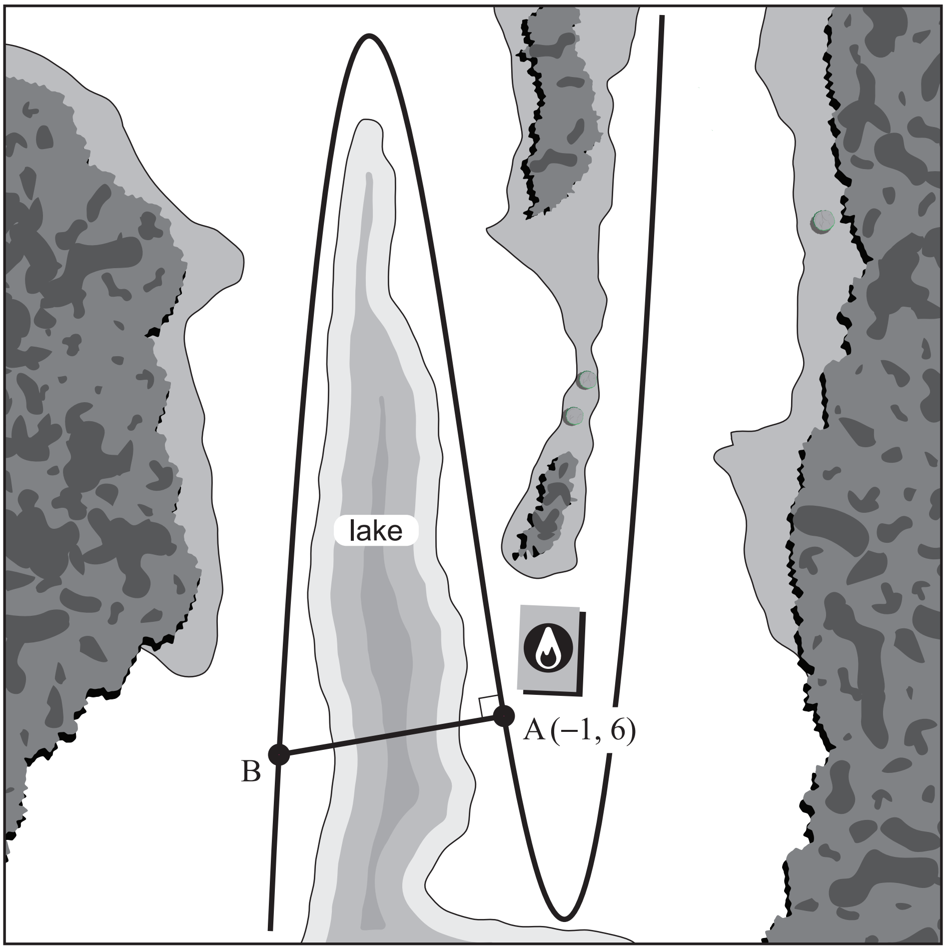
**8.** [Maximum mark: 7]

The diagram shows a map containing a long, winding road passing a lake. The shape of the road can be modelled by the function $r(x) = (x+1)^3 + 2x^2 - 4x$. All distances in the map are in kilometres.

The local fire station is located at point A, which has coordinates $(-1, 6)$.

To save time during emergencies, the local community is planning the construction of a bridge over the lake. The bridge will be built such that it is normal to the road at point A and will connect the fire station to point B.

**diagram not to scale**




(a) Using your graphic display calculator, find the value of $r'(-1)$. [2]

(b) Find the equation of the line normal to $r(x)$ at point A, which can be used to model the new bridge. [2]

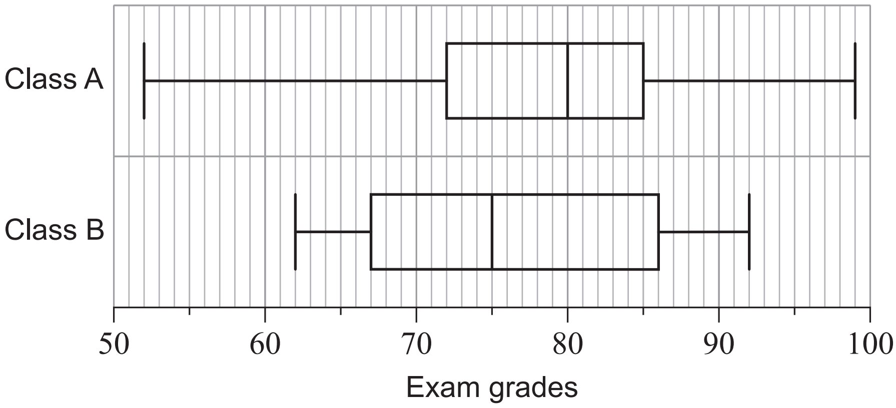
(c) Hence, determine the length of the new bridge. [3]

**(This question continues on the following page)**

**(Question 8 continued)**

**9.** [Maximum mark: 8]

Mr Kelly is a history teacher. After grading his final exams, he creates the following box and whisker diagram to compare the grades of his two classes.




(a) Identify which **two** of the following statements **must** be true according to the box and whisker diagram. Indicate your choices by placing tick marks in the second column of the following table. [2]

| **Statement** | **True (✓)** |
|---|---|
| A higher percentage of students in Class B received a grade less than 70 on the exam, than in Class A. | |
| The data for Class B is normally distributed. | |
| More students in Class A received a grade greater than 90 on the exam than in Class B. | |
| The interquartile range for Class A is less than the interquartile range for Class B. | |

At the end of the year, Mr Kelly surveyed a random sample of students from each of his two large classes to determine how satisfied they were with his teaching.

Each student independently selected a value from 1 to 10, with 1 meaning that they were not satisfied at all and 10 meaning that they were very satisfied.

His collected data from the student surveys is shown.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Class A** | 6 | 9 | 8 | 10 | 1 | 9 | 10 | 9 | 8 | 4 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Class B** | 7 | 5 | 3 | 4 | 3 | 8 | 6 | 7 |

**(This question continues on the following page)**

**(Question 9 continued)**

Mr Kelly believes that there was no difference in the general satisfaction between the two classes. He assumes that the data is drawn from a population that can be modelled by a normal distribution and proposes to conduct a $t$-test at the $5\%$ significance level.

(b) Write down the null and alternative hypotheses for his test. [2]

(c) Find the $p$-value for his test. [2]

(d) Write down the conclusion to the test. Give a reason for your answer. [2]

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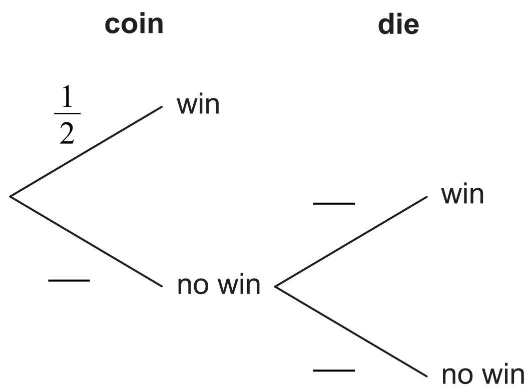
**10.** [Maximum mark: 7]

Michèle is playing a game. In the game, she must first flip a fair coin which will result in the coin landing on heads or tails. If the coin lands on heads, then she wins a prize. If it lands on tails, then she has another chance but this time she must roll a fair six-sided die and get a five or six in order to win a prize.

(a) Complete the tree diagram by writing in the three missing probabilities. [2]




(b) Find the probability that Michèle does **not** win a prize. [2]

(c) Given that Michèle won a prize, find the probability that the coin landed on heads. [3]

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**11.** [Maximum mark: 6]

Toktam works at a local bakery $5$ days each week. She drives an old car to work that has a $65\%$ probability of starting on any given morning. The probability of the car starting on a given morning is independent of it starting on any other morning.

(a) Find the probability that Toktam's car starts on exactly two mornings in a particular $5$ day workweek. [2]

Toktam walks to work on mornings when her car does not start and it is **not** raining. Toktam takes a taxi to work on mornings when her car does not start and it is raining.

Where Toktam lives, there is a $45\%$ probability of rain on any given morning, independent of any other morning. The probability of Toktam's car starting is independent of the weather.

(b) Find the probability that Toktam will **not** have to take a taxi in a particular workweek. [4]

**12.** [Maximum mark: 6]

Thurston believes that more popular musical artists sell more albums.

He begins to investigate this belief by randomly selecting eight musical artists and collecting data on the number of followers each of the artists has on a particular social media platform. He then collects data on the number of albums each artist sold in the first week after releasing an album. His data is shown in **Table 1**.

**Table 1**

| | **Artist 1** | **Artist 2** | **Artist 3** | **Artist 4** | **Artist 5** | **Artist 6** | **Artist 7** | **Artist 8** |
|---|---|---|---|---|---|---|---|---|
| **Number of social media followers (in thousands)** | 11 500 | 12 400 | 1300 | 2300 | 674 | 49 500 | 315 | 94 400 |
| **Number of albums sold in first week (in thousands)** | 123 | 62.4 | 17.4 | 94.9 | 52.5 | 27 | 21.6 | 595.5 |

Thurston decides to calculate the Spearman's rank correlation coefficient.

(a) Complete the table of ranks shown in **Table 2**. [1]

**Table 2**

| | **Artist 1** | **Artist 2** | **Artist 3** | **Artist 4** | **Artist 5** | **Artist 6** | **Artist 7** | **Artist 8** |
|---|---|---|---|---|---|---|---|---|
| **Rank – social media followers** | 4 | 3 | 6 | 5 | 7 | 2 | 8 | 1 |
| **Rank – albums sold in first week** | | | | | | | | 1 |

**(This question continues on the following page)**

**(Question 12 continued)**

(b) Calculate the value of $r_s$, Spearman's rank correlation coefficient. [2]

Thurston believes that artists with a higher number of social media followers sell more albums in the first week. He carries out a hypothesis test using a $10\%$ significance level with the following null hypothesis:

$H_0$: In the population, there is no monotonic relationship between the number of social media followers and the number of albums sold in the first week.

(c) Write down Thurston's alternative hypothesis. [1]

The critical value of $r_s$ for this test is $0.643$.

(d) State the conclusion of the hypothesis test, giving a reason. [2]

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**References:**

**2.** Nina Aldin Thune. https://en.wikipedia.org/wiki/Great_Pyramid_of_Giza#/media/File:Kheops-Pyramid.jpg. Licensed under CC BY 2.5 https://creativecommons.org/licenses/by/2.5/#. Image adapted.